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February 1998

Approved for public release; distribution unlimited.

IDA Paper P-3314

Log: H 97-001282

This work was conducted under contract DASW01 94 C 0054, Task T-G7-1353, for the Office of the Director, Acquisition Program Integration. The publication of this IDA document does not indicate endorsement by the Department of Defense, nor should the contents be construed as reflecting the official position of that Agency.

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PREFACE

This paper was prepared by the Institute for Defense Analyses (IDA) for the Office of the Director, Acquisition Program Integration, under a task entitled "Program Risk Analysis and Management." This paper partially fulfills that task by comparing the feasibility and costs of two procedures whereby contractors reveal probability distributions of cost while bidding on defense contracts.

This work was reviewed within IDA by Bruce R. Harmon, An-Jen Tai, and William P. Rogerson. The authors also acknowledge suggestions from Eric V. Denardo.

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I. INTRODUCTION

A. BACKGROUND

The purpose of this paper is to begin to think through how the government would use probability distributions of potential outcomes of projects in evaluating competitive bids and awarding contracts. The topic is motivated by the recent development of a model, Risk Analysis and Cost Management (RACM), by Lockheed Martin—see Goldberg (March 1996) for an evaluation of the model.

The structure of competitive bidding processes is an area of intense academic and practical interest. In the general economics literature, the creation and analysis of such processes lie within the field called "mechanism design" [see Fudenberg and Tirole (1991) for a general discussion and Laffont and Tirole (1993) for application of the concepts to procurement and regulation]. The recent auction of a portion of the frequency spectrum by the federal government, discussed in McMillan (1994) and in McAfee and McMillan (1996), could yield the government upwards of $20 billion.[1] More auctions of the frequency spectrum, with even greater amounts at stake, are being proposed. The design of the recent auction by the government was strongly influenced by game theory, and the behavior of the participants in the auction was actively informed by game-theoretic advice.

The central mechanism of the frequency spectrum auction is described by the name of the process: simultaneous ascending auction. Rules were structured to require the participants to make increasing financial commitments as the auction progressed, while allowing the participants to constantly revise alliances for the purpose of constructing efficient proposals. At each stage, the participants were offered a menu of choices across which they could allocate their bids. The participants did not fear being locked into losing positions because they had a great deal of flexibility throughout the process.

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1 The broadband personal communications services (PCS) auctions in blocks A through F have generated bids totaling $20 billion. However, actual revenues may ultimately fall short of this figure because small businesses (who received preferential discounts) are finding it difficult to finance their bids; Communications Daily's Washington Telecom Newswire (1997).

Sealed bid auctions are often structured so that the highest bidder receives the object at the price offered. One problem is that the highest bidder does not capture the difference between his true valuation and the true valuation of the next-highest bidder. This problem motivates all participants to bid low. However, in a sealed bid auction in which the highest bidder receives the object at the price offered by the next-highest bidder, there is no incentive to bid lower than the perceived value—the auction is efficient. This result is originally due to Vickrey (1961). The simultaneous ascending auction of the frequency spectrum was designed to maximize government revenue while preserving efficient bidder behavior.

Another newly emerging topic in the literature is in the area of alliances. Harsanyi and Selten (1988) address the fact that cooperative processes often result in sharing joint gains by non-cooperative processes—create the joint gains and then divide them by bargaining. This is the central problem of alliances and of competitive bids by teams. Brandenburger and Nalebuff (1996) give a wide range of practical examples of creating and sharing joint gains, and provide a framework for analysis of other situations. Brandenburger (1996) contributes to the understanding of strategic structures and shows the difficulty of estimating the "types" of one's competitors while they are estimating the "types" of their competitors, and so on.

Brams (1995) presents a structure wherein further consideration of Nash equilibria, from which a player may lose by changing but may gain if the opponent is then motivated to change, can lead to a reduced and preferable set of solutions. A solution which may seem to be an equilibrium (for instance, to the government and several bidders) may be inferior to another solution that can be found by further exploration of the set of feasible solutions.

Rogerson (1994) presents a survey of incentive structures in the defense industrial context. The present work on RACM falls within this domain, so the insights in the Rogerson paper should help to inform the prospective user of RACM for competitive bidding and government contracting.

B. SCOPE OF PAPER

We first discuss the general problem of the government and bidders using probability distributions of cost in the award and management of government procurements. We illustrate the components of a straightforward scheme for the government to evaluate bids expressed in terms of probability distributions of cost.

We then consider in some detail the bidding procedure from the game-theoretic point of view, treating both government and bidders, and exploring how all of the participants will behave as a function of how others behave and of how the government assesses the utility of the bids. We propose two government-bidder decision models that lead to the award of the contract in a manner similar to the Vickrey auction used in the sale of U.S. Treasury bonds, and to the simultaneous ascending auction used in the sale of the frequency spectrum. The objective of these procedures is to elicit honest bids in competitive bidding.

Our first procedure, called the complete information procedure, or outside-in procedure, requires each bidder to submit a number of alternative strategies and corresponding costs. The government then chooses the alternative that maximizes its own utility, subject to its being the alternative that maximizes the utility of the winning contractor while still winning the contract. (If the winning contractor has revealed an alternative with higher utility to the government but lower utility to itself, but this alternative is not required in order to outbid the other competitors, then the government does not choose this alternative.)

Our second procedure, called the sequential ascending procedure, or inside-out procedure, evolves in stages. It is exactly equal in outcome to the complete information procedure, but requires the bidders to reveal, and the government to evaluate, much less information. The procedure simply asks the bidders to submit a round of bids. They submit the bids with highest utility to themselves. The government chooses from among them the bid with highest utility to itself. Then as many bidders as can do so submit bids with higher utility to the government, and the highest utility to themselves subject to this constraint. Then the government chooses from among them the bid with highest utility to itself. Next the bidders have another opportunity, and so on. The procedure terminates when only one bidder is left. If that bidder has an alternative with higher utility to the government, but lower utility to itself, the bidder does not have to reveal it.

The remarkable feature of the second procedure is that it is identical in outcome to the first procedure, while involving far less information. We provide an example of the two equivalent procedures, document them, present a mathematical demonstration of their equivalence, and include computer programs that implement each of them.

In Appendix A we demonstrate the optimality of honest bids in competitive bidding when the winning bidder receives the object at the price offered by the second-highest bidder (the Vickrey auction). In Appendix B we show that our two procedures, the complete information procedure and the sequential ascending procedure, have equivalent outcomes. In Appendix C we give computer programs for the two procedures and present an example in which they yield the same optimal government-bidder behavior. Finally, in Appendix D we demonstrate that linear utility functions are not all interchangeable when the opportunity costs of project overruns are explicitly considered.

II. GENERAL DISCUSSION

This chapter describes the use of probability distributions of cost in defense procurement. It addresses the relationships among probability distributions of costs, utilities to the government, and the scoring process usually applied in the selection of winning bidders. It also addresses government-bidder interactions, speculating on how the procurement mechanism should be structured to induce the bidders to design their best processes and reveal them to the government, while the best bidder still captures the benefits of its net efficiencies over the second-best bidder.

The first problem confronting the government is how to deal with probability distributions of cost. The second problem is structuring the bidding process to achieve the most effective and efficient outcomes. The third problem is the evaluation of the advantages and disadvantages of working with probability distributions of cost in combination with sequential bidding processes.

A. PROBABILITY DISTRIBUTIONS OF COST, UTILITIES, AND SCORING IN CONTRACT AWARDS

1. Probability Distributions of Cost

Lockheed Martin developed the Risk Analysis and Cost Management (RACM) model for estimating the probability distribution of the cost of producing a military system. Given a detailed set of engineering and production activities, RACM estimates probability distributions of cost for each activity. RACM then combines these distributions to yield the probability distribution of the overall cost outcome associated with the set of proposed activities.

If contractors gave the government probability distributions of cost for a set of alternative activities by those contractors, how would the government use them?

2. Utilities from Probability Distributions of Cost

The government might care about only the expected cost for any contractor, in which case the government's preferences would be described as "risk-neutral." In that instance the expected cost would capture all of the relevant information, and there would

be no need for contractors to provide, or for the government to evaluate probability distributions of cost. However, it is more likely that the government cares about additional features of the cost distribution, such as the possible range, the standard deviation, the probability of particular high-cost outcomes, and so on. Preferences over these features of the cost distribution may be summarized with a utility function, which assigns a utility value to each possible cost outcome. Rather than simply calculating the expected cost, the government might calculate the expected utility; the latter is the sum of the utility values weighted by their respective probabilities of occurrence. The expected utility approach was originally developed by Von Neumann and Morgenstern, and is described, for example, in DeGroot (1970, chapter 7).

The choice of a utility function would be difficult. For instance, what should be the penalty assigned to a cost that is twice as much as the expected cost?

3. Standard Scoring

In evaluating proposals for procurement, the government typically breaks down the product or service to be procured into a number of characteristics. They might be, for instance, technical quality, production capability, cost, management practices and previous accomplishments. A maximum possible point score is assigned to each of the characteristics, perhaps adding to 100 points.

The government proposal evaluation team scores all of the characteristics of the competing contractors' proposals, obtaining a total score for each contractor. Presumably, the contract is awarded to the highest-scoring bidder. This might not necessarily be the bidder with the lowest cost.

4. Utilities from Probability Distributions of Cost and Standard Scoring

Standard scoring might be used to assign scores to probability distributions as well as to point estimates of cost. If so, the availability of probability distributions of cost might be thought of as more and better information.

The evaluators would essentially be confronted with the problem of assigning utilities to occurrences of events and computing expected utilities using probability distributions of cost. The evaluators might be given guidance—for example, mark down severely any proposal where there is a positive probability assigned to a cost event twice as high as the average cost. Alternatively, the evaluators might impose their own utilities

on the probability distributions, essentially declining to mark scores down or up on the basis of low-probability events.

B. GOVERNMENT-BIDDER INTERACTIONS

1. Government-Bidder Interactions and Vickrey Auctions

The most efficient sealed-bid auction or bidding process is one that awards the object being offered to the highest bidder at the price offered by the second-highest bidder. This structure induces all bidders to bid according to their honest valuations. (See Appendix A for a demonstration of the optimality of honest bidding in this auction.)

In a U.S. Treasury auction, where there is not a single object being sold but a number of bonds, the total quantity is usually too large to be purchased by one bidder. Instead, all of the bidders at or above the market-clearing price receive the amount of bonds they bid for at the market-clearing price, until the supply is exhausted. For instance, suppose there are 100 units for sale and the highest bidder offers $105 for each of 30 units, the second-highest bidder offers $103 for each of 50 units, and the third-highest bidder offers $101 for each of 40 units. Then the highest bidder receives 30 units, the second-highest bidder receives 50 units, and the third-highest bidder receives 20 units, each at a price of $101.

The latest version of this type of auction is the simultaneous ascending auction of the frequency spectrum. In this case there are a number of bidders for a number of parts of the spectrum. The auction proceeds in rounds, and at each round the bidders can form alliances and increase the bids, with more commitments made by the bidders at each stage, until the process concludes. The main advantages of this mechanism to the bidders are that they do not have to reveal information about themselves beforehand, they are not locked in early, and they can reorganize at each round to create efficiencies.

2. Government-Bidder Interactions with Probability Distributions of Cost

Suppose that there are a number of bidders for a government contract. A bidder might be a team—contractor and subcontractors. Each bidder can employ a selection of actions to produce the product. The government wishes to select the bidder with maximum utility to the government. The bidders wish to win the contract, subject to the winning bidder receiving the maximum utility possible and just barely winning the bid. (This is essentially equivalent to the winning bidder receiving the second bidder's price.)

The government wishes to elicit the actions and their probability distributions from the bidders to maximize the government's utility. The bidders do not wish to reveal to the government all of their possible actions and the implications of these actions. The bidders fear that the government will maximize its utility and award the contract to the highest-ranking bidder based on that bidder's costs. In this case the highest-ranking bidder would not capture the surplus of its efficiency advantage over the second-highest-ranking bidder; rather, the government would capture that surplus. Unless the bidders can guard against this situation, they will not have incentives to bid honestly. Instead, they will have incentives to overstate the cost to the government of acquiring the goods or services being sought.

A variation of the simultaneous ascending auction should succeed in solving the government's problem of ensuring honest evaluation, while also easing the bidders' concern of the government capturing the profits due to the competitive efficiencies of the highest-ranking bidder.

3. What Does the Government Gain from Probability Distributions of Cost?

The current system deals in point estimates of costs. These point estimates are alleged to be "too conservative" in that the products should not cost as much as the bids; the contractors are adding too many safety factors, seeking to avoid losses.

If the government were to receive probability distributions of cost, it could award the contract in full knowledge that the product might cost more, or less, than the bid price. The government would be able to assign its own utilities to the probabilistic outcomes. However, the government and the contractors should have a mechanism to handle the low-probability events of higher costs or lower costs should they occur.

4. Government-Bidder Incentives After Contract Award

If the government were to receive a probability distribution of cost from a contractor and sign a contract, would there be an incentive for the contractor to allow the low-probability event of much higher-than-expected costs to occur?

The reason there would not be such an incentive is that the contract would be structured so that if the low-probability event of much-higher costs occurred, the profit to the contractor would be small or negative. Similarly, if the low-probability event of much-lower costs occurred, the profit to the contractor would be larger than planned.

The incentive for the contractor to misstate or hide information during the procurement award process in the latter case is reduced by the bidding process. If a bidder does not reveal information about much-lower-cost potential activities as the process unfolds, the chances of winning the contract are reduced.

C. ADVANTAGES AND DISADVANTAGES TO THE GOVERNMENT OF SEQUENTIAL BIDDING PROCESSES WITH PROBABILITY DISTRIBUTIONS

Suppose that the government simply asked the bidders for a single bid, to be evaluated before the contract was awarded. Suppose further that the evaluation procedure consisted of weighting and scoring various parts of the bid, with the contract awarded at a fixed price to the bidder with the highest weighted score. In this situation, there would be incentives for the contractors to overstate the cost to the government. These incentives would arise in part due to contractors hedging against their own uncertainties. In addition, contractors would want to avoid being awarded the contract unless they received at least a portion of the efficiency margin between their winning bid and the second-highest bid.

If the contract were awarded at cost plus fixed fee, the bidders might overstate the cost to the government to hedge against their own uncertainties. However, the probability of a bidder receiving the contract might increase if the bidder understated costs. The problem of efficiency margins discussed in the previous paragraph would not be as evident because any winner would receive a fee. The more the fee would be incentive-based, the more the contract would resemble a fixed-price contract.

The incentives would seem to be consistent if *all* of the following conditions held:

- the bidders submitted probability distributions of cost;
- the government could score these probability distributions;
- the government selected the most efficient bidder (as in a one-stage Vickrey auction to induce honest bids); and
- after the contract began, the review and award procedures would penalize low performance and reward high performance (presumably both low-probability events).

A sequential procedure might not induce the bidders to reveal as much information as would a one-stage procedure. Surprisingly, however, a sequential procedure can be designed that is essentially equivalent to having the contractors reveal all of their possible actions and probability distributions. We demonstrate in Appendix B that a one-stage "outside-in" procedure—where all actions are revealed—and a sequential

"inside-out" procedure—where only some actions are revealed—result in the same chosen bidder and the same chosen action.

The advantage of the sequential procedure is that the contractor need only reveal, and the government need only process, a much smaller amount of information. For instance, if there are six bidders with five actions each, the one-stage procedure requires 30 sets of data. In a particular numerical example that we explore, the corresponding sequential procedure requires six sets on the first round, three sets on the second round, two sets on the third round, and one set on the fourth round—a total of twelve sets of data. If the government would be required to evaluate 30 probability distributions with a one-stage procedure, but could elicit essentially all of the possible truthful information by evaluating only 12 probability distributions in a sequential procedure, the latter process would seem quite attractive.

III. GOVERNMENT DECISION MODELS

A. COST MODEL

Define:

c_i = cost at level i ($i = 1, \ldots, I$),

p_{ib} = probability of cost at level i for bidder b ($i = 1, \ldots, I$; $b = 1, \ldots, B$).

The expected cost for bidder b is:

$$E(c^b) = \sum_{i=1}^{I} c_i p_{ib}, \quad b = 1, \ldots, B.$$

Define:

U_i = government's utility of cost at level i, where $0 < U_i < 1$.

The government's expected utility from selecting bidder b is:

$$E(U^b) = \sum_{i=1}^{I} U_i p_{ib}, \quad b = 1, \ldots, B.$$

On the basis of expected cost or utility, the government would choose the bidder b^*, yielding minimum expected cost or maximum expected utility as follows:

$$E(c^*) = \min_b \left\{ E(c^b), b = 1, \ldots, B \right\},$$

$$E(U^*) = \max_b \left\{ E(U^b), b = 1, \ldots, B \right\}.$$

B. COST AND QUALITY MODEL

Define:

d_j = quality at level j ($j = 1, \ldots, J$),

q_{ijb} = probability of cost at level i and quality at level j for bidder b ($i = 1, \ldots, I$; $j = 1, \ldots, J$; $b = 1, \ldots, B$).

The expected cost for bidder b is:

$$E(c^b) = \sum_{i=1}^{I} \sum_{j=1}^{J} c_i q_{ijb}, \quad b = 1, \ldots, B.$$

The expected quality for bidder b is:

$$E(d^b) = \sum_{i=1}^{I} \sum_{j=1}^{J} d_j q_{ijb}, \quad b = 1, \ldots, B.$$

Define:

V_{ij} = government's utility of cost at level i and quality at level j, where $0 < V_{ij} < 1$.

The government's expected utility from selecting bidder b is:

$$E(V^b) = \sum_{i=1}^{I} \sum_{j=1}^{J} V_{ij} q_{ijb}, \quad b = 1, \ldots, B.$$

On the basis of expected cost, quality, or utility, the government would choose the bidder b^*, yielding minimum expected cost, maximum expected quality, or maximum expected utility as follows:

$$E(c^*) = \min_b \left\{ E(c^b), b = 1, \ldots, B \right\},$$

$$E(d^*) = \max_b \left\{ E(d^b), b = 1, \ldots, B \right\},$$

$$E(V^*) = \max_b \left\{ E(V^b), b = 1, \ldots, B \right\}.$$

C. COST, QUALITY, AND TIME MODEL

Define:

e_k = time at level k ($k = 1, \ldots, K$),

r_{ijkb} = probability of cost, quality, and time at respective levels i, j, k for bidder b ($i = 1, \ldots, I; j = 1, \ldots, J; k = 1, \ldots, K; b = 1, \ldots, B$).

The expected cost for bidder b is:

$$E(c^b) = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} c_i r_{ijkb}, \quad b = 1, \ldots, B.$$

The expected quality for bidder b is:

$$E(d^b) = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} d_j r_{ijkb}, \quad b = 1, \ldots, B.$$

The expected time for bidder b is:

$$E(e^b) = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} e_k r_{ijkb}, \quad b = 1, \ldots, B.$$

Define:

W_{ijk} = government's utility of cost at level i, quality at level j, and time at level k, where $0 < W_{ijk} < 1$.

The government's expected utility from selecting bidder b is:

$$E(W^b) = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} W_{ijk} r_{ijkb}, \quad b = 1, \ldots, B.$$

On the basis of expected cost, quality, time, or utility, the government would choose the bidder b^*, yielding minimum expected cost, maximum expected quality, minimum expected time, or maximum expected utility as follows:

$$E(c^*) = \min_b \left\{ E(c^b), b = 1, \ldots, B \right\},$$

$$E(d^*) = \max_b \left\{ E(d^b), b = 1, \ldots, B \right\},$$

$$E(e^*) = \min_b \left\{ E(e^b), b = 1, \ldots, B \right\},$$

$$E(W^*) = \max_b \left\{ E(W^b), b = 1, \ldots, B \right\}.$$

D. COST MODEL SAMPLE APPLICATION

1. Utility Measures

The utility measure of the government can be of virtually any form for which utility decreases as cost increases. For instance, suppose that the budgeted cost is 50, with utility 0.50, and that for costs at Levels 1 through 9, there are four utility measures as follows:

Table III-1. Utility Measures

		Utility Measure			
Level	Cost	1	2	3	4
1	10	.90	.70	.70	.90
2	20	.80	.65	.65	.80
3	30	.70	.60	.60	.70
4	40	.60	.55	.55	.60
5	50	.50	.50	.50	.50
6	60	.40	.45	.40	.45
7	70	.30	.40	.30	.40
8	80	.20	.35	.20	.35
9	90	.10	.30	.10	.30

Utility Measure 1 is linear and steep. Utility Measure 2 is linear and shallow. Interestingly, these two utility measures are equivalent because they both represent risk-neutral preferences.[1] As we will see shortly, a linear utility measure provides the same ranking of alternatives as does a simple calculation of expected cost.

Utility Measure 3 is linear and shallow for cost savings and linear and steep for cost overruns. Utility Measure 4 is linear and steep for cost savings and linear and shallow for cost overruns. Utility Measure 3 can be thought of as reflecting risk-aversion, whereas Utility Measure 4 can be thought of as reflecting risk-preference.

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1 The equivalence holds only in the narrow sense of selecting among alternative bidders or strategies to achieve the same outcome. A broader context would consider the allocation between a large project with a risk of over-running, and the small projects that might be squeezed out of a fixed investment budget. We demonstrate in Appendix D that, within that broader context, the slope of the utility function (the "marginal utility of income") plays a key role in determining the project portfolio.

2. Example with Two Bidders

a. Base Case Probability Distributions

Assume that there are two bidders with probability distributions of cost as follows:

Table III-2. Probability Distributions of Cost

Level	Cost	Probability Distributions Bidder 1	Bidder 2
1	10	0.0	.05
2	20	0.0	.05
3	30	0.0	.05
4	40	0.0	.05
5	50	1.0	.60
6	60	0.0	.05
7	70	0.0	.05
8	80	0.0	.05
9	90	0.0	.05

Assume that the utility measures are as given in the previous section.

Note that for all levels of cost, except when cost equals 50, Utility Measure 3 is less than Utility Measure 4. The utility of achieving lower costs is less and the utility of suffering higher costs is also less—Utility Measure 3 undervalues cost savings and abhors cost overruns and thus is lower than Utility Measure 4, except at the midpoint. So for a particular bidder Utility Measure 3 will always be lower than Utility Measure 4. However, assuming that the government has just one utility function, the more interesting question is how that utility function ranks the various bidders. Of course, the ranking might change depending on the particular utility function we adopt.

Expected costs and expected utilities for the four measures are as follows:

Table III-3. Expected Costs and Expected Utilities

	Bidder 1	Bidder 2
Cost	50	50
Utility 1	.500	.500
Utility 2	.500	.500
Utility 3	.500	.475
Utility 4	.500	.525

For expected cost as a measure, the government is indifferent between Bidder 1 and Bidder 2. The government is indifferent between the two bidders under Utility Measures 1 and 2 as well. Both of these utility measures are linear (albeit with different slopes) representing risk-neutrality, in which case the only pertinent feature of the cost distribution is its expected value. For non-linear Utility Measures 3 and 4, however, risk becomes pertinent. If the government is risk-averse (as in Utility Measure 3), it prefers the certainty of Bidder 1, but if it values risk (as in Utility Measure 4), it prefers Bidder 2.

b. First Variation of Probability Distribution of Bidder 1

Assume that there are two bidders with probability distributions as follows, where the only change from the above data is that Bidder 1's probability shifts upward one level:

Table III-4. First Variation of Probability Distribution of Cost

		Probability Distributions	
Level	Cost	Bidder 1	Bidder 2
1	10	0.0	.05
2	20	0.0	.05
3	30	0.0	.05
4	40	0.0	.05
5	50	0.0	.60
6	60	1.0	.05
7	70	0.0	.05
8	80	0.0	.05
9	90	0.0	.05

Expected costs and expected utilities for the four measures are as follows:

Table III-5. Expected Costs and Expected Utilities for First Variation

	Bidder 1	Bidder 2
Cost	60	50
Utility 1	.400	.500
Utility 2	.450	.500
Utility 3	.400	.475
Utility 4	.450	.525

For expected cost as a measure, Bidder 2 is now preferred by a margin of 10. Bidder 2 is also preferred by all four utility measures. Although Utility Measure 3 still

reflects risk-aversion, Bidder 2 is now preferred over Bidder 1 because the 10-unit margin in expected cost more than compensates for the greater risk of Bidder 2.

c. Second Variation of Probability Distribution of Bidder 1

Assume that there are two bidders with probability distributions as follows, where the change from the base case is that Bidder 1's probability is divided equally among costs at Levels 3 through 7, or costs from 30 to 70, as follows:

Table III-6. Second Variation of Probability Distribution of Cost

		Probability Distributions	
Level	Cost	Bidder 1	Bidder 2
1	10	0.0	.05
2	20	0.0	.05
3	30	0.2	.05
4	40	0.2	.05
5	50	0.2	.60
6	60	0.2	.05
7	70	0.2	.05
8	80	0.0	.05
9	90	0.0	.05

Expected costs and expected utilities for the four measures are as follows:

Table III-7. Expected Costs and Expected Utilities for Second Variation

	Bidder 1	Bidder 2
Cost	50	50
Utility 1	.500	.500
Utility 2	.500	.500
Utility 3	.470	.475
Utility 4	.530	.525

For expected cost as a measure, the government is indifferent between Bidder 1 and Bidder 2. With expected cost again equalized, risk-neutral Utility Measures 1 and 2 also indicate indifference. Unlike the previous variation, both bidders now embody risk. Risk-averse Utility Measure 3 selects Bidder 2, which has more probability concentrated near the mean of the cost distribution though some probability in the extreme tails. Conversely, risk-preferring Utility Measure 4 selects Bidder 1, which has less probability concentrated near the mean.

IV. GOVERNMENT-BIDDER DECISION MODEL—EXAMPLE

A. MODEL FOR EXAMPLE

Define strategies f_i and g_j of Bidders 1 and 2. Strategy f_i generates distribution x_i and strategy g_j generates distribution y_j. Suppose that the government maps distribution x_i into utility u_i and distribution y_j into utility v_j. Distributions and utilities are as follows, with the maximum utility of the government for Bidders 1 and 2 having values u^* and v^*, respectively.

Table IV-1. Distributions

	Bidder 2		
Bidder 1	g_1	g_2	g_3
f_1	x_1, y_1	x_1, y_2	x_1, y_3
f_2	x_2, y_1	x_2, y_2	x_2, y_3
f_3	x_3, y_1	x_3, y_2	x_3, y_3

Table IV-2. Utility Matrix

	Bidder 2		
Bidder 1	g_1	g_2	g_3
f_1	u_1, v_1	u_1, v_2	u_1, v_3
f_2	u_2, v_1	u_2, v_2	u_2, v_3
f_3	u_3, v_1	u_3, v_2	u_3, v_3
$u^* = \max(u_1, u_2, u_3)$			
$v^* = \max(v_1, v_2, v_3)$			

Suppose that expected profits of the bidders are as follows, with the maximum profit of Bidder 1 and Bidder 2 denoted by p^* and q^*, respectively.

Table IV-3. Profit Matrix

Bidder 1	Bidder 2		
	g_1	g_2	g_3
f_1	p_1, q_1	p_1, q_2	p_1, q_3
f_2	p_2, q_1	p_2, q_2	p_2, q_3
f_3	p_3, q_1	p_3, q_2	p_3, q_3
$p^* = \max(p_1, p_2, p_3)$			
$q^* = \max(q_1, q_2, q_3)$			

Suppose that the government chooses Bidder 1 corresponding to $w^* = \max(u^*, v^*)$. Suppose that w^* corresponds to f_3 and that $p_3 < p^*$. Should Bidder 1 withdraw f_3 and submit just f_1 and f_2?

To answer this question, we now analyze the reduced matrix (without f_3):

Table IV-4. Reduced Utility Matrix

Bidder 1	Bidder 2		
	g_1	g_2	g_3
f_1	u_1, v_1	u_1, v_2	u_1, v_3
f_2	u_2, v_1	u, v_2	u_2, v_3
$u^* = \max(u_1, u_2)$			
$v^* = \max(v_1, v_2, v_3)$			

Suppose that the government chooses Bidder 2 corresponding to $w^* = \max(u^*, v^*)$. Suppose that w^* corresponds to g_3 and that $q_3 < q^*$. Should Bidder 2 withdraw g_3 and submit just g_1 and g_2?

To answer this question, we now analyze the further reduced matrix (without f_3 or g_3):

Table IV-5. Further Reduced Utility Matrix

Bidder 1	Bidder 2	
	g_1	g_2
f_1	u_1, v_1	u_1, v_2
f_2	u_2, v_1	u_2, v_2
$u^* = \max(u_1, u_2)$		
$v^* = \max(v_1, v_2)$		

Suppose that the government chooses Bidder 2 corresponding to $w^* = \max(u^*, v^*)$ and that w^* corresponds to g_2. Suppose that $q^* \geq q_2 > q_3$. In this case Bidder 2 would be wise to drop g_3 because it won the bid and increased its profit from q_3 to q_2. But Bidder 1, knowing that Bidder 2 would eventually win the bid with g_2, by the above logic, would not be willing to withdraw f_3 in the first place.

B. SUMMARY OF EXAMPLE

Bidders define strategies f and g that result in distributions x and y, government utilities u and v, and bidder profits p and q. If the government were to choose Bidder 1 based on that bidder's strategy f_3, Bidder 1 still might wish to withdraw f_3 if it could improve its profit. However, if Bidder 1 withdrew f_3 and the government chose Bidder 2 based on g_3, Bidder 1 would be disappointed. Furthermore, could Bidder 2 then withdraw g_3, retain the contract and improve its profit? The example traces through this chain of reasoning, and motivates further analysis of the optimal behavior of Bidder 1 and Bidder 2.

V. TWO EQUIVALENT GOVERNMENT-BIDDER DECISION MODELS

A. COMPLETE INFORMATION PROCEDURE (OUTSIDE-IN)

1. Assumptions

Assume that the bidders all put forward a number of actions for consideration by the government. The logic of this choice procedure is that the government first has the opportunity to choose the bidder and action of that bidder to maximize its utility over all of the bidders and their respective actions. The bidder who will be chosen first then (conceptually) withdraws the winning alternative. The government then has the opportunity to maximize its utility over the reduced set. If the same bidder is still chosen, and if its utility is equal to or greater than the earlier utility, the bidder is better off. The bidder then (conceptually) withdraws the second winning alternative, and so on, until it is no longer chosen. It then returns to its last winning alternative.

2. Definitions

Define the following notation:

a_{ij} = bidder i action j,

x_{ij} = distribution of outcomes for bidder i action j,

u_{ij} = utility of government for x_{ij},

v_{ij} = utility of bidder i for x_{ij}.

Also define the following relationships:

$$x_{ij} = f_i(a_{ij}),$$

where $f_i[\,]$ is the production process of bidder i;

$$u_{ij} = g[f_i(a_{ij})],$$

where $g[\,]$ is the utility function of the government; and

$$v_{ij} = h_i[f_i(a_{ij})],$$

where $h_i[\,]$ is the utility function of bidder i.

3. Procedure

For a_{ij} compute x_{ij}, u_{ij}, and v_{ij} ($i = 1, \ldots, I; j = 1, \ldots, J$).

a. Step 1

Let the set of I times J alternatives be denoted by IJ_1. The government chooses $ij{*}1$ such that:

$$u_{ij*1} = \max_{\{ij \in IJ_1\}} u_{ij} .$$

Denote the winning bidder by $i{*}1$. The utility to the government is u_{ij*1}, and the utility to the winning bidder is v_{ij*1}.

b. Step 2

The winning bidder eliminates alternative $ij{*}1$. Let the new set of alternatives be denoted by IJ_2. The government chooses $ij{*}2$ such that:

$$u_{ij*2} = \max_{\{ij \in IJ_2\}} u_{ij} .$$

Denote the winning bidder by $i{*}2$. The utility to the government is u_{ij*2}, and the utility to the winning bidder is v_{ij*2}. If $i{*}2$ is different from $i{*}1$, then $i{*}1$ accepts Step 1 and the results are those of Step 1. If $i{*}2$ is the same as $i{*}1$ then $i{*}2$ accepts Step 2 if $v_{ij*2} > v_{ij*1}$; otherwise, it accepts Step 1.

c. Step 3

The winning bidder eliminates alternative $ij{*}2$. Let the new set of alternatives be denoted by IJ_3. The government chooses $ij{*}3$ such that:

$$u_{ij*3} = \max_{\{ij \in IJ_3\}} u_{ij} .$$

Denote the winning bidder by $i{*}3$. The utility to the government is u_{ij*3}, and the utility to the winning bidder is v_{ij*3}. If $i{*}3$ is different from $i{*}2$, then $i{*}2$ accepts Step 2 and the results are those of Step 2. If $i{*}3$ is the same as $i{*}2$ then $i{*}3$ accepts Step 3 if $v_{ij*3} > v_{ij*2}$; otherwise, it accepts Step 2.

d. **Step 4 and Subsequent Steps**

Continue until the winning bidder changes.

B. SEQUENTIAL ASCENDING PROCEDURE (INSIDE-OUT)

1. Introduction

We show in Appendix B that this procedure leads to the same result as the outside-in procedure. The inside-out procedure is similar in nature to the simultaneous ascending auction described in McMillan (1994) and in McAfee and McMillan (1996). Assume that exactly the same information is present as for the outside-in procedure. This time, however, the information is used differently.

2. Procedure

a. Step 1

For $i = 1, \ldots, I$, let each bidder reveal the action j that maximizes its utility over v_{ij} ($j = 1, \ldots, J$). Call this utility v_{i1} with associated utility to the government u_{i1}. The government chooses a bidder such that:

$$u_{i*1} = \max_{\{i\}} u_{i1} .$$

The winning bidder is $i*1$ and the utility to the government is u_{i*1}.

b. Step 2

Each bidder observes the winning bid from Step 1 and the identity of the winning bidder, $i*1$. Each bidder then submits the bid with highest v_{ij} subject to $u_{i2} > u_{i*1}$, if it can meet this constraint; otherwise, it does not submit any bid at this step. If $i*1$ is the only remaining bidder, the procedure ends with Step 1 above. If other bidders remain, then the government chooses a bidder such that:

$$u_{i*2} = \max_{\{i\}} u_{i2} .$$

The winning bidder is $i*2$ and the utility to the government is u_{i*2}.

c. Step 3

Each bidder observes the winning bid from Step 2 and the identity of the winning bidder, $i*2$. Each bidder then submits the bid with highest v_{ij} subject to $u_{i3} > u_{i*2}$, if it can meet this constraint; otherwise, it does not submit any bid at this step. If $i*2$ is the only remaining bidder, the procedure ends with Step 2 above. If other bidders remain, then the government chooses a bidder such that:

$$u_{i*3} = \max_{\{i\}} u_{i3} .$$

The winning bidder is $i*3$ and the utility to the government is u_{i*3}.

d. Step n+1

Continue until no bids are available such that $u_{i,n+1} > u_{i*n}$. At this point the winning bidder is $i*n$. The action $j*n$ corresponding to $i*n$ is $a_{i*n,\, j*n}$. The utility to the government is $u_{i*n,\, j*n}$. The utility to the winning bidder is $v_{i*n,\, j*n}$.

C. EXAMPLE

Assume that there are three bidders, each with three actions, and assume the utilities for the government and for the bidders are as follows:

Table V-1. Government Utilities

	Action 1	Action 2	Action 3
Bidder 1	50	61	51
Bidder 2	20	70	65
Bidder 3	60	10	40

Table V-2. Bidder Utilities

	Action 1	Action 2	Action 3
Bidder 1	50	39	49
Bidder 2	80	30	35
Bidder 3	40	90	60

1. Complete Information Procedure (Outside-In)

(1) Government selects Bidder 2, Action 2, yielding utility 70 to the government and utility 30 to Bidder 2.

(2) Bidder 2 withdraws Action 2. Government selects Bidder 2, Action 3, yielding utility 65 to the government and utility 35 to Bidder 2.

(3) Bidder 2 withdraws Action 3. Government selects Bidder 1, Action 2, yielding utility 61 to the government and utility 39 to Bidder 1.

Result: The procedure terminates with step (2), yielding utility 65 to the government and utility 35 to Bidder 2.

2. Sequential Ascending Procedure (Inside-Out)

(1) Bidder 1 offers Action 1 with utilities 50 and 50 to the government and the bidder, respectively.

Bidder 2 offers Action 1 with utilities 20 and 80, respectively.

Bidder 3 offers Action 2 with utilities 10 and 90, respectively.

Government selects Bidder 1, Action 1.

(2) Bidder 1 offers Action 3 with utilities 51 and 49, respectively.

Bidder 2 offers Action 3 with utilities 65 and 35, respectively.

Bidder 3 offers Action 1 with utilities 60 and 40, respectively.

Government selects Bidder 2, Action 3.

(3) Bidder 1 has no offer.

Bidder 2 offers Action 2 with utilities 70 and 30, respectively.

Bidder 3 has no offer.

Result: The procedure terminates with step (2), yielding utility 65 to the government and utility 35 to Bidder 2.

Notice that the two procedures yield the same outcome. The generality of this result is proved in Appendix B.

VI. EFFECTS OF PARTICIPATION COSTS

A. INTRODUCTION

This chapter investigates the effects on utility to the government and utility to the bidders of participation costs in the sequential ascending procedure. It shows how the sequence of choices and the final outcome change as assumptions about participation costs change.

Fewer steps may occur before completion if participation costs are included, and the successful bidder at each step may change, as well as the winning bidder at the final step.

Stegeman (1996) studies the effect of participation costs on the efficiency of auctions and concludes that a second-price auction is efficient when participation costs are present. The structures he studies and our sequential ascending procedure are similar but not identical, so it is not known whether or not our procedure is efficient when there are participation costs. This chapter explores the topic and illustrates the impact of participation costs in a particular, fairly rich, example.

B. EXAMPLE PROBLEM AND SOLUTION

The example problem investigated here is the same as the example problem of Appendix C. There are six bidders with five actions each. Utilities to the government and bidders are as follows:

Table VI-1. Utility to Government

	Action				
Bidder	1	2	3	4	5
1	50	61	51	48	58
2	40	70	65	77	87
3	60	55	90	92	40
4	91	93	44	94	90
5	75	90	66	40	50
6	25	45	77	55	65

Table VI-2. Utility to Bidders

	Action				
Bidder	1	2	3	4	5
1	50	61	51	65	75
2	20	75	65	72	82
3	60	10	59	45	65
4	30	25	65	24	32
5	20	33	80	60	70
6	30	33	77	75	85

The sequential procedure results in four steps, as follows:

Table VI-3. Results of Sequential Ascending Procedure

			Utilities	
Step	Bidder	Action	Government	Bidder
1	2	5	87	82
2	3	3	90	59
3	3	4	92	45
4	4	2	93	25
Final Result	4	2	93	25

In Step 1, all six bidders put forward the proposals with highest utility to them; Bidder 2 is best with utility to the government of 87. In Step 2, three bidders can beat the Step 1 utility to the government of 87. This step is won by Bidder 3, who offers utility to the government of 90 with utility to Bidder 3 of 59; we break the tie based on the latter utility, which exceeds the utility to either Bidder 4 or Bidder 5 were they to choose actions that yield utility to the government of 90. In Step 3, two bidders can beat the Step 2 utility to the government of 90; Bidder 3 is again best with 92. In Step 4 only one bidder can beat the Step 3 utility to the government of 92; Bidder 4 is best with 93.

C. VARIATIONS IN PARTICIPATION COSTS

1. Penalties as a Percentage of Utilities

In the first variation, a penalty is assessed to the government and to the bidders beginning with step 2. The penalty is a percentage of the utility remaining at each step.

Results are as follows for 1-, 2-, and 3-percent penalties:

Table VI-4. Penalty of 1 Percent

			Utilities	
Step	Bidder	Action	Government	Bidder
1	2	5	87.00	82.00
2	3	3	89.10	58.41
3	3	4	90.17	44.10
4	4	2	90.24	24.26
Final Result	4	2	90.24	24.26

Table VI-5. Penalty of 2 Percent

			Utilities	
Step	Bidder	Action	Government	Bidder
1	2	5	87.00	82.00
2	3	3	88.20	57.82
3	4	2	89.32	24.01
4	4	2	87.53	23.53
Final Result	4	2	87.53	23.53

Table VI-6. Penalty of 3 Percent

			Utilities	
Step	Bidder	Action	Government	Bidder
1	2	5	87.00	82.00
2	3	3	87.30	57.23
3	4	2	87.50	23.52
4				
Final Result	4	2	87.50	23.52

For 1-percent penalties, the optimal procedure is the same as in the base case. For 2-percent penalties, the optimal procedure chooses Bidder 4 at the third step over Bidder 3 (who also submits an improved bid relative to Step 2); the choice of Bidder 4 is made final at the fourth step. For 3-percent penalties, the procedure chooses Bidder 4 at the third step and, because no other bidders exceed the utility of Step 2, the procedure terminates.

Optimal utilities decrease as penalties increase. When the penalty is 1 percent, the optimal sequence is the same as in the base case. However, the three extra steps yield a utility to the government of 90.24, a 3.0-percent decrease in utility compared to the base

case. When the penalty is 2 percent, the three extra steps yield a utility to the government of 87.53, a 5.9-percent decrease compared to the base case. Changing from a 2-percent penalty to a 3-percent penalty results in a utility to the government of 87.50. Although the optimal sequence changes, the final utility to the government is essentially the same as in the 2-percent case.

2. Penalties That Are Fixed and Identical

In the second variation, the penalty to the government and the bidders is an absolute number per step beginning with step 2.

Results are as follows for penalties of 1, 2, and 3:

Table VI-7. Penalty of 1

			Utilities	
Step	Bidder	Action	Government	Bidder
1	2	5	87	82
2	3	3	89	58
3	4	2	91	23
4	4	2	90	22
Final Result	4	2	90	22

Table VI-8. Penalty of 2

			Utilities	
Step	Bidder	Action	Government	Bidder
1	2	5	87	82
2	3	3	88	57
3	4	2	89	21
4				
Final Result	4	2	89	21

Table VI-9. Penalty of 3

			Utilities	
Step	Bidder	Action	Government	Bidder
1	2	5	87	82
2	3	4	89	42
3	3	4	86	39
4				
Final Result	3	4	86	39

For a penalty of 1, the optimal procedure chooses Bidder 4 at the third step and confirms the choice at the fourth step. For a penalty of 2, the optimal procedure chooses Bidder 4 at the third step and, because no other bidders exceed the utility of Step 2, the procedure terminates. For a penalty of 3, the optimal procedure chooses Bidder 3 at the second step and confirms the choice at the third step. This is the first variation of the base case that terminates with a result other than Bidder 4, Action 2.

Optimal utilities decrease as penalties increase. When the penalty is 1 the optimal utility is 90, or 3 less than the base case. When the penalty is 2 the optimal utility is 89, or 4 less than the base case; termination is earlier than when the penalty is 1. When the penalty is 3 the optimal utility is 86; the choice of Bidder 3 must be confirmed in the third step. Note that the utility to Bidder 3 of 39 far exceeds the utility to Bidder 4 of the previous variations.

3. Penalties That Are Fixed and Different

In the third variation, the penalty to the government is smaller than to the bidders. Specifically, the penalty to the government is half of the penalty to the bidders. Results are as follows for respective penalties of 0.5 and 1.0, 1.0 and 2.0, and 1.5 and 3.0:

Table VI-10. Penalties of 0.5 and 1.0

			Utilities	
Step	Bidder	Action	Government	Bidder
1	2	5	87.0	82.0
2	3	3	89.5	58.0
3	3	4	91.0	43.0
4	4	2	91.5	22.0
Final Result	4	2	91.5	22.0

Table VI-11. Penalties of 1.0 and 2.0

			Utilities	
Step	Bidder	Action	Government	Bidder
1	2	5	87.0	82.0
2	3	3	89.0	57.0
3	4	2	91.0	21.0
4	4	2	90.0	19.0
Final Result	4	2	90.0	19.0

Table VI-12. Penalties of 1.5 and 3.0

Step	Bidder	Action	Utilities: Government	Utilities: Bidder
1	2	5	87.0	82.0
2	3	3	88.5	56.0
3	4	2	90.0	19.0
4	4	2	88.5	16.0
Final Result	4	2	88.5	16.0

For the first set of penalties, the optimal procedure is the same as the base case. For the second set, the optimal procedure chooses Bidder 4 at the third step and confirms the choice at the fourth step. For the third set of penalties, the optimal procedure is the same as for the second set.

Optimal utilities to the government decrease as the penalties increase, from 93.0 in the base case to 91.5 for the first set of penalties, 90.0 for the second set, and 88.5 for the third. Optimal utilities for the bidders decrease from 25.0 in the base case to 22.0, 19.0, and 16.0 for the three sets of penalties.

D. INSIGHTS FOR EXAMPLE PROBLEM

In this example, only a few variations have participation costs that affect the winner finally selected. The participation costs do, however, affect the optimal sequence in many of the variations. Also, large participation costs can have a significant effect on the final utilities to the government and the bidders.

One basic issue is the practical extent of participation costs. Let us assume that participation costs are included in the cost to the government. The government would directly pay its own participation costs as well as those incurred by the winning bidder. It might also pay indirectly, through overhead charges, some or all of the participation costs incurred by the losing bidders.

If this example is representative, and if the participation costs were relatively small, they would probably not affect either the final winner or the utilities to the government and the bidders. The sequential ascending procedure, which is closely related to the second-price auction, could be followed and preserve the advantages to all parties.

If the participation costs were relatively large, they might affect the final winner and the utilities to the government and bidders. The government might have to pay not only its own costs, but also those of both the winner and the losers. Consideration of participation costs would then be important, and these costs might outweigh the benefits of the sequential procedure relative to the one-stage procedure.

VII. CONCLUSIONS

A. PROBABILITY DISTRIBUTIONS OF COST, UTILITIES, AND INCENTIVES

This paper explores the use of probability distributions of cost in competitive bidding for government procurement contracts. An attempt is made to identify advantages and disadvantages of using probability distributions, both to the government and to the bidders. The behavioral incentives of all parties are also investigated.

A key issue, assuming that the bidders are able to furnish honest estimates of probability distributions of cost, is how the government can cope with these probability distributions. If the government could develop utility functions, it could compute the expected utility associated with any probability distribution, select the bidder that provides the highest expected utility, and thus improve upon a selection based on only expected costs submitted by bidders.

It should be possible to modify standard scoring methods used in awarding contracts to handle probability distributions of cost. An interesting question is whether each of the evaluators on a government selection team would furnish his or her own utility function, or whether a higher authority would furnish the utility function (and, if so, how that utility function would be determined by the higher authority).

The incentive for bidders to misstate information during the bidding process might be reduced by a requirement to submit probability distributions. The contract would probably be structured so that if the low-probability event of a much lower-than-expected cost occurred, the contractor would receive higher profits; while if the low-probability event of a much higher-than-expected cost occurred, the contractor would receive lower profits.

B. SEQUENTIAL BIDDING PROCESSES

If the contract were awarded to the bidder with the highest expected utility to the government at the cost estimated by the lowest bidder, there might be incentives for all bidders to overstate costs. If the contract were awarded to the bidder with the highest

expected utility to the government at the cost estimated by the *second-lowest* bidder, as in the Vickrey auction, the incentives to estimate cost honestly would be improved.

We designed a sequential bidding procedure that allows bidders to reveal less information, and the government to process less information, while still inducing honest bidding. The sequential procedure has the same winning bidder, with the same utility to the government and to the bidder, as a one-stage procedure requiring the bidders to furnish and the government to evaluate much more information.

This paper develops both a one-stage procedure and a sequential procedure. It describes the procedures, gives proofs of their equivalence, furnishes computer programs, and presents examples.

C. PARTICIPATION COSTS

The costs incurred by the government in evaluating bids are a significant aspect of the government contracting process. So, too, are the costs incurred by the bidders, although these costs are at least partially absorbed by the government. It is desirable to maximize the utility to the government, and to induce honest bidding, in order to maximize the overall utility to society.

This paper shows that the utilities to the government and the bidders are sensitive to the participation costs in a sequential ascending procedure. In some of our examples, the course of the sequential procedure is unchanged by the presence of participation costs, while in other examples both the winning bidder and the action taken by that bidder are different. We do not offer a general conclusion on this matter, but show how to explore the effects of alternative assumptions about participation costs and the sensitivity of the procedure to these costs.

D. NEED FOR FURTHER WORK

The paper addresses how the government would use probability distributions of cost instead of single point estimates. The government has two related motivations for considering this question. First, the government cares about more than just the mean of the cost distribution. Second, the government is concerned that bidders may not have proper incentives to truthfully reveal even the mean. Requiring the bidders to submit entire probability distributions instead of just point estimates might provide the government with greater leverage for the elicitation of truthful information.

The paper first describes methods for comparing probability distributions of cost assuming that bidders could be induced to truthfully reveal them. It then addresses the establishment of proper incentives for truthful revelation. The incentives take the form of two different, but ultimately equivalent, bidding procedures. Of the two, the sequential procedure yields the same outcome, but requires the disclosure and processing of less information.

The incentive structures considered here are limited. One way to make more progress on the topic might be to try to relate it to the literature of menus of contracts. For example, bidders could bid both a cost and a sharing ratio. This type of bid would require knowledge of and perhaps communication of information about probability distributions of cost. The analytical structure would attempt to incorporate explicitly how cost-sharing provisions of the contract awarded to the winner could be chosen as a function of the nature of the bids.

The one-stage procedure and the sequential procedure in this paper can perhaps serve as prototypes for a competition in which each bidder formulates multiple strategies, each with an associated probability distribution of cost. Using the procedures given here, the strategies could be evaluated either simultaneously or sequentially.

With respect to the specific procedures analyzed in this paper, we do not explicitly treat how probability distributions of cost are translated into bids. Rather, we assume that the government and bidders can assess the utility of each bid based on a number of characteristics, including probability distribution of cost.

APPENDIX A

OPTIMALITY OF HONEST BIDS IN COMPETITIVE BIDDING WITH A SECOND-PRICE AWARD

A seller has one indivisible unit of an object for sale. There are I potential bidders. The bidders simultaneously submit bids. The highest bidder wins the object and pays the second bid price.

Define for bidders $i = 1, ..., I$:

v_i = valuation of bidder i for the object,

s_i = bid of bidder i for the object,

$r_i = \underset{j \neq i}{Max}\, s_j$ (highest bid other than s_i),

$u_i = v_i - r_i$ = utility of winning bid for bidder i.

The optimal strategy for bidder i is to bid exactly his valuation, or $s_i = v_i$. Bidder i will obtain utility 0 (for a non-winning bid) or utility $v_i - r_i$ (for a winning bid). The three cases of bidding too high ($s_i > v_i$) and the three cases of bidding too low ($s_i < v_i$) are analyzed below. We show that there is no advantage to anything but an honest bid.

$s_i > v_i$: Bidder i bids more than his valuation

1. $r_i > s_i > v_i$ (bidder i does not win the object). Bidder i obtains utility 0, which he would also have obtained by bidding v_i.
2. $s_i > v_i > r_i$ (bidder i wins the object). Bidder i obtains utility $v_i - r_i$, which he would also have obtained by bidding v_i.
3. $s_i > r_i > v_i$ (bidder i wins the object). Bidder i obtains the negative utility $v_i - r_i < 0$; if he had bid v_i, his utility would have been 0.

$s_i < v_i$: Bidder *i* bids less than his valuation

1. $r_i < s_i < v_i$ (bidder *i* wins the object). Bidder *i* obtains utility $v_i - r_i$, which he would also have obtained by bidding v_i.
2. $s_i < v_i < r_i$ (bidder *i* does not win the object). Bidder *i* obtains utility 0, which he would also have obtained by bidding v_i.
3. $s_i < r_i < v_i$ (bidder *i* does not win the object). Bidder *i* obtains utility 0; if he had bid v_i, his utility would have been $v_i - r_i > 0$.

QED

APPENDIX B

EQUIVALENCE OF COMPLETE INFORMATION PROCEDURE AND SEQUENTIAL ASCENDING PROCEDURE

OUTSIDE-IN AUCTION

Assume bidder i^* is the one with the potential offer that globally maximizes the government's utility:

$$u_{i^* j^*} = \underset{\substack{i \in I \\ j \in J}}{Max}\, u_{ij}\,. \tag{B–1}$$

Bidder i^* is guaranteed to win the outside-in auction, because no other bidder can exceed its initial offer of (i^*, j^*). (We assume the absence of "ties".) However, bidder i^* can improve its own position, while still winning the auction, by choosing a strategy j' possibly different from j^*. Specifically, bidder i^* will choose strategy j' that solves:

$$v_{i^*,j'} = \underset{\{j \in J\}}{Max}\, v_{i^*,j} \quad \text{subject to} \quad u_{i^*,j'} > \underset{\substack{i \neq i^* \\ j \in J}}{Max}\, u_{i,j}\,. \tag{B–2}$$

A bit more can be said if we impose the additional constraint of a constant-sum game:

$$u_{ij} + v_{ij} = \text{constant}, \quad \forall i \in I, j \in J\,. \tag{B–3}$$

In this case, the increase in the bidder's utility is matched by an equal decrease in the government's utility, so that strategy j' also solves:

$$u_{i^*,j'} = \underset{\{j \in J\}}{Min}\, u_{i^*,j} \quad \text{subject to} \quad u_{i^*,j'} > \underset{\substack{i \neq i^* \\ j \in J}}{Max}\, u_{i,j}\,. \tag{B–4}$$

INSIDE-OUT AUCTION

Bidder i^* is also guaranteed to win the inside-out auction. Suppose the contrary, that some bidder $i' \neq i^*$ had the winning bid at step n of the auction. Bidder i^* can find at least one strategy [for example, (i^*, j^*)] that yields higher utility to the government than

any strategy that bidder i' could offer. Thus, by the rules of the inside-out auction, bidder i^* must win.

When tendering this superior offer, bidder i^* will choose the particular strategy that maximizes its own utility, subject to out-bidding all of its rivals. Thus, the winning strategy again solves problem (B–2). Further, under the constraint of a constant-sum game, the winning strategy also solves problem (B–4).

QED

APPENDIX C

COMPUTER PROGRAM FOR TWO EQUIVALENT GOVERNMENT-BIDDER DECISION MODELS

This appendix contains a computer program that solves an example problem where six bidders have five potential actions each. Utilities to the government and bidders are as follows:

Table C-1. Utility to Government

	Action				
Bidder	1	2	3	4	5
1	50	61	51	48	58
2	40	70	65	77	87
3	60	55	90	92	40
4	91	93	44	94	90
5	75	90	66	40	50
6	25	45	77	55	65

Table C-2. Utility to Bidders

	Action				
Bidder	1	2	3	4	5
1	50	61	51	65	75
2	20	75	65	72	82
3	60	10	59	45	65
4	30	25	65	24	32
5	20	33	80	60	70
6	30	33	77	75	85

ONE-STAGE PROCEDURE AND FINAL RESULT

The one-stage procedure, or "outside-in" procedure, requires three steps. The outcomes of each step are as follows: (1) bidder selected, (2) action of bidder selected, (3) utility to government, and (4) utility to bidder.

Table C-3. Results of One-Stage Procedure

Step	Bidder	Action	Utilities: Government	Utilities: Bidder
1	4	4	94	24
2	4	2	93	25
3	3	4	92	45
Final Result	4	2	93	25

Bidder 4 wins the contract with Action 2, with utilities to the government and bidder of 93 and 25, respectively.

SEQUENTIAL PROCEDURE AND FINAL RESULT

The sequential procedure, or "inside-out" procedure, has four steps. In Step 1, all six bidders put forward their proposals with highest utility to them; Bidder 2 is best with utility to the government of 87. In Step 2, three bidders can beat the Step 1 utility to the government of 87. We break the tie by awarding this step to Bidder 3, who offers utility to the government of 90 and utility to Bidder 3 of 59 (we break the tie based on the latter, which exceeds the utility to either Bidder 4 or Bidder 5 were they to choose actions that also yield utility to the government of 90). In Step 3, two bidders can beat the Step 2 utility to the government of 90; Bidder 3 is again best with 92. In Step 4, only one bidder can beat the Step 3 utility to the government of 92; Bidder 4 is best with 93.

Table C-4. Results of Sequential Procedure

Step	Bidder	Action	Utilities: Government	Utilities: Bidder
1	2	5	87	82
2	3	3	90	59
3	3	4	92	45
4	4	2	93	25
Final Result	4	2	93	25

The remainder of this appendix contains the FORTRAN code for a computer program that solves the example.

PROGRAM GBO

```
C
C --- MAIN PROGRAM
C
 1      FORMAT(1H )
 10     FORMAT(10I8)
 20     FORMAT(10F8.0)
22      FORMAT(10F8.2)
C
        DIMENSION U(6,5),V(6,5)
C
        OPEN(6,FILE='6.OUT',STATUS='NEW')
C
C       IT=3
C       JT=3
C
        DATA U /     50.,40.,60.,91.,75.,25.,
        X            61.,70.,55.,93.,90.,45.,
        X            51.,65.,90.,44.,66.,77.,
        X            48.,77.,92.,94.,40.,55.,
        X            58.,87.,40.,90.,50.,65. /
C
        DATA V /     50.,20.,60.,30.,20.,30.,
        X            61.,75.,10.,25.,33.,33.,
        X            51.,65.,59.,65.,80.,77.,
        X            65.,72.,45.,24.,60.,75.,
        X            75.,82.,65.,32.,70.,85. /
C
        IT=6
        JT=5
C
        WRITE(6,1)
        DO 110 I=1,IT
 110    WRITE(6,20)(U(I,J),J=1,JT)
C
        WRITE(6,1)
```

```
      DO 120 I=1,IT
120   WRITE(6,20)(V(I,J),J=1,JT)
C
      CALL GBO1(U,V,IT,JT,IF,JF,UF,VF)
C
      WRITE(6,1)
      WRITE(6,1)
      WRITE(6,10)IF,JF
      WRITE(6,20)UF,VF
C
      CALL GBO2(U,V,IT,JT,IF,JF,UF,VF)
C
      WRITE(6,1)
      WRITE(6,1)
      WRITE(6,10)IF,JF
      WRITE(6,20)UF,VF
C
      STOP
      END
```

```
SUBROUTINE GBO1(U,V,IT,JT,IF,JF,UF,VF)
C
C --- ONE-STAGE PROCEDURE
C
C --- COMPLETE INFORMATION PROCEDURE (OUTSIDE IN)
C
 1      FORMAT(1H )
 10     FORMAT(10I8)
 20     FORMAT(10F8.0)
 22     FORMAT(10F8.2)
 30     FORMAT(2I10,2F10.0)
C
        DIMENSION U(6,5),V(6,5)
C
        WRITE(6,1)
        WRITE(6,101)
 101    FORMAT(1X,'GBO1')
C
C --- STEP 1
C
        UMAX=0.
        VMAX=0.
        DO 150 I=1,IT
        DO 150 J=1,JT
              IF(U(I,J)-UMAX)150,147,140
 140                UMAX=U(I,J)
                    VMAX=V(I,J)
                    IMAX=I
                    JMAX=J
                    GO TO 150
 147          IF(V(I,J)-VMAX)150,150,148
 148                VMAX=V(I,J)
                    IMAX=I
                    JMAX=J
 150    CONTINUE
C
```

```
      IS1=IMAX
      JS1=JMAX
      US1=UMAX
      VS1=VMAX
C
      WRITE(6,1)
      WRITE(6,30)IS1,JS1,US1,VS1
C
C --- STEP 2 -- SEE IF SAME BIDDER MAXIMIZES U AS IN STEP 1
C               IF NO, SOLUTION IS PREVIOUS BID
C               IF YES AND V EQUAL OR GREATER, CONTINUE
C
      UMAX=0.
      VMAX=V(I,J)
      DO 290 I=1,IT
      DO 290 J=1,JT
            IF(I-IS1)280,202,280
202         IF(J-JS1)280,290,280
C
280         IF(U(I,J)-UMAX)290,287,285
285               UMAX=U(I,J)
                  VMAX=V(I,J)
                  IMAX=I
                  JMAX=J
                  GO TO 290
287         IF(V(I,J)-VMAX)290,290,288
288               VMAX=V(I,J)
                  IMAX=I
                  JMAX=J
290   CONTINUE
C
      IS2=IMAX
      JS2=JMAX
      US2=UMAX
      VS2=VMAX
C
```

```
      WRITE(6,1)
      WRITE(6,30)IS2,JS2,US2,VS2
C
      IF(IS2-IS1)296,295,296
295   IF(VS2-VS1)296,300,300
296   ISEND=1
      GO TO 1000
C
C --- STEP 3 -- SEE IF SAME BIDDER MAXIMIZES U AS IN STEP 2, ETC
C
300   UMAX=0
      VMAX=0.
      DO 390 I=1,IT
      DO 390 J=1,JT
            IF(I-IS1)311,302,311
302         IF(J-JS1)311,390,311
311         IF(I-IS2)380,312,380
312         IF(J-JS2)380,390,380
C
380         IF(U(I,J)-UMAX)390,387,385
385               UMAX=U(I,J)
                  VMAX=V(I,J)
                  IMAX=I
                  JMAX=J
                  GO TO 390
387         IF(V(I,J)-VMAX)390,390,388
388               VMAX=V(I,J)
                  IMAX=I
                  JMAX=J
390   CONTINUE
C
      IS3=IMAX
      JS3=JMAX
      US3=UMAX
      VS3=VMAX
C
```

```
      WRITE(6,1)
      WRITE(6,30)IS3,JS3,US3,VS3
C
      IF(IS3-IS2)396,395,396
395   IF(V(IS3,JS3)-V(IS2,JS2))396,400,400
396         ISEND=2
      GO TO 1000
C
C --- STEP 4 -- SEE IF SAME BIDDER MAXIMIZES U AS IN STEP 3, ETC
C
400   UMAX=0.
      VMAX=0.
      DO 490 I=1,JT
      DO 490 J=1,JT
C
            IF(I-IS1)411,402,411
402         IF(J-JS1)411,490,411
C
411         IF(I-IS2)421,412,421
412         IF(J-JS2)421,490,421
C
421         IF(I-IS3)480,422,480
422         IF(J-JS3)480,490,480
C
480         IF(U(I,J)-UMAX)490,487,485
485               UMAX=U(I,J)
                  VMAX=V(I,J)
                  IMAX=I
                  JMAX=J
                  GO TO 490
487         IF(V(I,J)-VMAX)490,490,488
488               VMAX=V(I,J)
                  IMAX=I
                  JMAX=J
490   CONTINUE
C
```

```
      IS4=IMAX
      JS4=JMAX
      US4=UMAX
      VS4=VMAX
C
      WRITE(6,1)
      WRITE(6,30)IS4,JS4,US4,VS4
C
      IF(IS4-IS3)496,495,496
 495  IF(V(IS4,JS4)-V(IS3,JS3))496,500,500
 496        ISEND=3
      GO TO 1000
 500 CONTINUE
C
C --- END
C
 1000 IF(ISEND-2)1010,1020,1030
C
 1010 IF=IS1
            JF=JS1
            UF=US1
            VF=VS1
      GO TO 1100
C
 1020 IF=IS2
            JF=JS2
            UF=US2
            VF=VS2
      GO TO 1100
C
 1030 IF=IS3
            JF=JS3
            UF=US3
            VF=VS3
      GO TO 1100
C
```

```
1100  RETURN
      END
```

```
SUBROUTINE GBO2(U,V,IT,JT,IF,JF,UF,VF)
C
C --- SEQUENTIAL PROCEDURE
C
C --- PARTIAL INFORMATION PROCEDURE (INSIDE OUT)
C
 1      FORMAT(1H )
 10     FORMAT(10I8)
 20     FORMAT(10F8.0)
 22     FORMAT(10F8.2)
 30     FORMAT(2I10,2F10.0)
C
        DIMENSION U(6,5),V(6,5)
C
        DIMENSION VCAND(6),JCAND(6),UCAND(6)
C
        WRITE(6,1)
        WRITE(6,91)
 91     FORMAT(1X,'GBO2')
C
C --- STEP 1
C
        WRITE(6,1)
        WRITE(6,101)
 101    FORMAT(1X,'STEP 1')
C
C --- BIDDERS -- CHOOSE BID WHICH MAXIMIZES V
C
        DO 150 I=1,IT
                VMAX=0.
                DO 140 J=1,JT
                        IF(V(I,J)-VMAX)140,140,135
135                             VMAX=V(I,J)
                                JMAX=J
140             CONTINUE
                VCAND(I)=VMAX
```

```
            JCAND(I)=JMAX
            UCAND(I)=U(I,JMAX)
 150    CONTINUE
        WRITE(6,1)
        WRITE(6,10)(JCAND(I),I=1,IT)
        WRITE(6,20)(VCAND(I),I=1,IT)
        WRITE(6,20)(UCAND(I),I=1,IT)
C
C --- GOVERNMENT -- CHOOSE BID WHICH MAXIMIZES U GIVEN V
C
        UMAX=0.
        VMAX=0.
        DO 160 I=1,IT
            IF(UCAND(I)-UMAX)160,157,155
 155            UMAX=UCAND(I)
                VMAX=VCAND(I)
                IMAX=I
                GO TO 160
 157        IF(VCAND(I)-VMAX)160,160,158
 158            VMAX=VCAND(I)
                IMAX=I
 160    CONTINUE
C
        IS1=IMAX
        JS1=JCAND(IMAX)
        US1=UMAX
        VS1=VCAND(IMAX)
C
        WRITE(6,1)
        WRITE(6,30)IS1,JS1,US1,VS1
C
C --- STEP 2
C
 200    WRITE(6,1)
        WRITE(6,201)
 201    FORMAT(1X,'STEP 2')
```

```
C
C - BIDDERS - CHOOSE BID WHICH MAXIMIZES V SUBJECT TO EXCEEDING U
C
      ITER=0
      UGOAL=US1
 202  DO 250 I=1,IT
         UCAND(I)=0.
         JCAND(I)=0
         VCAND(I)=0.
         VGOAL=0.
         DO 240 J=1,JT
            IF(U(I,J)-UGOAL)240,240,230
 230        IF(V(I,J)-VGOAL)240,240,235
 235           UCAND(I)=U(I,J)
               JCAND(I)=J
               VCAND(I)=V(I,J)
               VGOAL=VCAND(I)
 240     CONTINUE
 250  CONTINUE
      WRITE(6,1)
      WRITE(6,10)(JCAND(I),I=1,IT)
      WRITE(6,20)(VCAND(I),I=1,IT)
      WRITE(6,20)(UCAND(I),I=1,IT)
C
C --- GOVERNMENT -- CHOOSE BID WHICH MAXIMIZES U GIVEN V
C
      UMAX=0.
      VMAX=0.
      IMAX=0
      DO 260 I=1,IT
         IF(UCAND(I)-UMAX)260,257,255
 255        UMAX=UCAND(I)
            VMAX=VCAND(I)
            IMAX=I
            GO TO 260
 257     IF(VCAND(I)-VMAX)260,260,258
```

```
 258                  VMAX=VCAND(I)
                      IMAX=I
 260    CONTINUE
C
        IF(IMAX-0)261,261,262
C
 261           IS2=0
               JS2=0
               US2=0.
               VS2=0.
               GO TO 263
C
 262           IS2=IMAX
               JS2=JCAND(IMAX)
               US2=UMAX
               VS2=VCAND(IMAX)
C
 263    WRITE(6,1)
        WRITE(6,30)IS2,JS2,US2,VS2
C
C --- TERMINATION TEST:
C --- 1. IF THERE IS NOT A SUCCESSFUL BIDDER, END WITH PREVIOUS BEST BID
C --- 2. IF THERE IS ONE SUCCESSFUL BIDDER AND IT IS THE PREVIOUS
C ---  BIDDER, END WITH PREVIOUS BEST BID
C --- 3. OTHERWISE, CONTINUE
C
        IF(IS2-0)295,295,280
C
C        BETTER BIDDER, SEE IF PREVIOUS
C
 280    IF(IS2-IS1)300,285,300
C
C --- BIDDER PREVIOUS, SEE IF ONLY
C
 285    NSUC=0
        DO 290 I=1,IT
```

```
            IF(JCAND(I)-0)290,290,286
 286        NSUC=NSUC+1
 290    CONTINUE
C
        IF(NSUC-1)295,295,300
C
C --- BIDDER PREVIOUS AND ONLY
C
 295    ISEND=1
        GO TO 1000
C
C --- STEP 3
C
 300    WRITE(6,1)
        WRITE(6,301)
 301    FORMAT(1X,'STEP 3')
C
C -BIDDERS - CHOOSE BID WHICH MAXIMIZES V SUBJECT TO EXCEEDING U
C
        ITER=0
        UGOAL=US2
 302    DO 350 I=1,IT
            UCAND(I)=0.
            JCAND(I)=0
            VCAND(I)=0.
            VGOAL=0.
            DO 340 J=1,JT
                IF(U(I,J)-UGOAL)340,340,330
 330            IF(V(I,J)-VGOAL)340,340,335
 335                UCAND(I)=U(I,J)
                    JCAND(I)=J
                    VCAND(I)=V(I,J)
                    VGOAL=VCAND(I)
 340        CONTINUE
 350    CONTINUE
        WRITE(6,1)
```

```
      WRITE(6,10)(JCAND(I),I=1,IT)
      WRITE(6,20)(VCAND(I),I=1,IT)
      WRITE(6,20)(UCAND(I),I=1,IT)
C
C --- GOVERNMENT -- CHOOSE BID WHICH MAXIMIZES U GIVEN V
C
      UMAX=0.
      VMAX=0.
      IMAX=0
      DO 360 I=1,IT
          IF(UCAND(I)-UMAX)360,357,355
355           UMAX=UCAND(I)
              VMAX=VCAND(I)
              IMAX=I
              GO TO 360
357       IF(VCAND(I)-VMAX)360,360,358
358           VMAX=VCAND(I)
              IMAX=I
360   CONTINUE
C
      IF(IMAX-0)361,361,362
C
361       IS3=0
          JS3=0
          US3=0.
          VS3=0.
          GO TO 363
C
362       IS3=IMAX
          JS3=JCAND(IMAX)
          US3=UMAX
          VS3=VCAND(IMAX)
C
363   WRITE(6,1)
      WRITE(6,30)IS3,JS3,US3,VS3
C
```

```
C --- TERMINATION TEST:
C --- 1. IF THERE IS NOT A SUCCESSFUL BIDDER, END WITH PREVIOUS BEST BID
C --- 2. IF THERE IS ONE SUCCESSFUL BIDDER AND IT IS THE PREVIOUS
C ---  BIDDER, END WITH PREVIOUS BEST BID
C --- 3. OTHERWISE, CONTINUE
C
      IF(IS3-0)395,395,380
C
C      BETTER BIDDER, SEE IF PREVIOUS
C
 380  IF(IS3-IS2)400,385,400
C
C --- BIDDER PREVIOUS, SEE IF ONLY
C
 385  NSUC=0
      DO 390 I=1,IT
           IF(JCAND(I)-0)390,390,386
 386       NSUC=NSUC+1
 390  CONTINUE
C
      IF(NSUC-1)395,395,400
C
C --- BIDDER PREVIOUS AND ONLY
C
 395  ISEND=2
      GO TO 1000
C
C --- STEP 4
C
 400  WRITE(6,1)
      WRITE(6,401)
 401  FORMAT(1X,'STEP 4')
C
C - BIDDERS - CHOOSE BID WHICH MAXIMIZES V SUBJECT TO EXCEEDING U
C
      ITER=0
```

```
      UGOAL=US3
 402  DO 450 I=1,IT
         UCAND(I)=0.
         JCAND(I)=0
         VCAND(I)=0.
         VGOAL=0.
         DO 440 J=1,JT
            IF(U(I,J)-UGOAL)440,440,430
 430        IF(V(I,J)-VGOAL)440,440,435
 435           UCAND(I)=U(I,J)
               JCAND(I)=J
               VCAND(I)=V(I,J)
               VGOAL=VCAND(I)
 440     CONTINUE
 450  CONTINUE
      WRITE(6,1)
      WRITE(6,10)(JCAND(I),I=1,IT)
      WRITE(6,20)(VCAND(I),I=1,IT)
      WRITE(6,20)(UCAND(I),I=1,IT)
C
C --- GOVERNMENT -- CHOOSE BID WHICH MAXIMIZES U GIVEN V
C
      UMAX=0.
      VMAX=0.
      IMAX=0
      DO 460 I=1,IT
         IF(UCAND(I)-UMAX)460,457,455
 455        UMAX=UCAND(I)
            VMAX=VCAND(I)
            IMAX=I
            GO TO 460
 457     IF(VCAND(I)-VMAX)460,460,458
 458        VMAX=VCAND(I)
            IMAX=I
 460  CONTINUE
C
```

```
      IF(IMAX-0)461,461,462
C
 461          IS4=0
              JS4=0
              US4=0.
              VS4=0.
              GO TO 463
C
 462          IS4=IMAX
              JS4=JCAND(IMAX)
              US4=UMAX
              VS4=VCAND(IMAX)
C
 463  WRITE(6,1)
      WRITE(6,30)IS4,JS4,US4,VS4
C
C --- TERMINATION TEST:
C --- 1. IF THERE IS NOT A SUCCESSFUL BIDDER, END WITH PREVIOUS BEST BID
C --- 2. IF THERE IS ONE SUCCESSFUL BIDDER AND IT IS THE PREVIOUS
C ---  BIDDER, END WITH PREVIOUS BEST BID
C --- 3. OTHERWISE, CONTINUE
C
      IF(IS4-0)495,495,480
C
C      BETTER BIDDER, SEE IF PREVIOUS
C
 480  IF(IS4-IS3)500,485,500
C
C --- BIDDER PREVIOUS, SEE IF ONLY
C
 485  NSUC=0
      DO 490 I=1,IT
              IF(JCAND(I)-0)490,490,486
 486          NSUC=NSUC+1
 490  CONTINUE
C
```

```
      IF(NSUC-1)495,495,500
C
C --- BIDDER PREVIOUS AND ONLY
C
 495  ISEND=3
      GO TO 1000
C
C --- END WITH STEP 4 RESULTS
C
500   IF=IS4
      JF=JS4
      UF=US4
      VF=VS4
      GO TO 1100
C
 1000 IF(ISEND-2)1010,1020,1030
C
 1010         IF=IS1
              JF=JS1
              UF=US1
              VF=VS1
              GO TO 1100
C
 1020 IF=IS2
              JF=JS2
              UF=US2
              VF=VS2
              GO TO 1100
 1030 IF=IS3
              JF=JS3
              UF=US3
              VF=VS3
              GO TO 1100
C
 1100 RETURN
      END
```

APPENDIX D

LINEAR UTILITY AND THE MARGINAL UTILITY OF INCOME

We observed in Chapter III that all linear utility functions, regardless of their slope, imply risk neutrality. A risk-neutral decision-maker ranks alternative projects in terms of expected cost only, with no consideration of the variance or other higher-order moments of the cost distribution. Thus, it appears that all linear utility functions are interchangeable.

In this appendix, we demonstrate that linear utility functions are not all interchangeable when the opportunity costs of project overruns are explicitly considered. Instead, the slope of the utility function (the "marginal utility of income") plays an important role in determining the allocation between a large project with a risk of overrunning, and the small projects that might be squeezed out of a fixed investment budget.

In the following section of this appendix, we develop an interpretation of the marginal utility of income. In the second section, we exhibit the interplay between the marginal utility of income, and the decision-maker's willingness to secure additional funding for small projects in the face of an overrun on a large project.

SENSITIVITY OF THE PROJECT PORTFOLIO TO THE MARGINAL UTILITY OF INCOME

Society has a money endowment of \$$E$. There are two types of projects, large and small. At most one large project is physically possible. But multiple, identical small projects may be undertaken. For example, a large project may be to build an anti-missile defense system for Washington, DC. A small project may be to pave 100 miles of highway. The latter project may be replicated many times.

Each project has a cost: C_l for the large project, and C_s for each small project. Income equals the endowment, \$$E$, less the total cost of all projects funded. Utility depends on a linear function of income remaining for other uses, I. Utility also depends

on the numbers of large and small projects funded (to capture their non-monetary benefits):

$$U = f\left(\alpha + \beta I, \#\text{large}, \#\text{small}\right),$$

where $\beta > 0$.

Suppose that we fund one large project and k small projects. The resulting utility level equals:

$$U = f\left[\alpha + \beta\left(E - C_1 - kC_s\right), 1, k\right]. \tag{D–1}$$

Let f_I denote the partial derivative of the utility function in its first argument, and let f_k denote its partial derivative in the third argument. The optimal number of small projects may be determined by setting to zero the derivative of the utility function with respect to k. The optimal number satisfies the following first-order condition:

$$0 = -\beta C_s f_I + f_k. \tag{D–2}$$

The optimal value of k is a function of the parameter β, the marginal utility of income. We will demonstrate that $\partial k/\partial\beta < 0$. This result is intuitive, because β measures the importance of reserving income for uses other than funding the two types of projects under consideration. When the sentiment to reserve income is smaller, society will dig deeper into its pockets and fund more small projects.

To demonstrate this result, we will argue by analogy with the canonical model of consumer demand in microeconomic theory. Consider a utility function $U = f(x, k)$ defined over two goods x and k. Suppose that the unit prices are p_x and p_k, while total income equals M. Because the price of x will not change throughout this analysis, we lose no generality by redefining the unit of measure, if necessary, so that $p_x = 1$.[1] Then the consumer must maximize utility subject to the constraint:

$$x + p_k k = M.$$

---

1 If, for example, $p_x = 0.25$, redefine a new unit of x as four of the old units. Then $p_x = 1.00$.

Substituting this constraint into the utility function, the consumer must maximize:

$$f(M - p_k k, k).$$

The optimal value of k satisfies the following first-order condition:

$$0 = -p_k f_x + f_k. \tag{D–3}$$

The optimal value of k is a function of its price, p_k. Comparing equations (D–2) and (D–3), we see that the term βC_s in equation (D–2) plays the same role as the price p_k in equation (D–3). It is generally assumed that demand curves are negatively sloped so that, other factors held constant, $\partial k/\partial p_k < 0$. According to Henderson and Quandt (1980, equation 2-30, p. 26), this condition is equivalent to a certain restriction on the second partial derivatives of the utility function.[2] If we are willing to accept this restriction as being reasonable, then when applied to utility function (D–1) we have $\partial k/\partial(\beta C_s) < 0$. Finally, because C_s is fixed, it follows that $\partial k/\partial \beta < 0$.

THE MARGINAL UTILITY OF INCOME AND THE RESPONSE TO COST OVERRUNS

We have seen that the marginal utility of income β is an inverse measure of the importance of funding small projects. This parameter also determines society's response to a cost overrun on the large project. Suppose that, for a base case, we fund one large project and k_1 small projects. The base utility level equals:

$$U_{\text{base}} = f\left[\alpha + \beta\left(E - C_1 - k_1 C_s\right), 1, k_1\right].$$

Now we experience a cost overrun on the large project: cost increases from C_1 to C_2. If we were to keep total investment expenditures constant, we could afford only a reduced number of small projects, k_2, where:

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2 Interestingly, concavity of the utility function is neither necessary nor sufficient to ensure a negatively sloped demand curve. To see that concavity is not sufficient, consider the following concave utility function: $U = -(2-x)^4/(k-1)^2$ for $x < 2$ and $k > 1$. Although this utility function is concave, the demand for k is upward sloping when $Max\{2, p_k + 1\} < M < p_k + 2$.

$$k_2 \;=\; Max\left[0, k_1 - \frac{(C_2 - C_1)}{C_s}\right] \;<\; k_1 .$$

Assume that k_2 is a positive integer. With investment expenditures constant, income remaining for other uses is *unchanged*. But utility still falls because society loses the non-monetary benefits of the small projects that are squeezed out:

$$\begin{aligned} U_{\text{overrun}} &= f\left[\alpha + \beta\left(E - C_2 - k_2 C_s\right), 1, k_2\right] \\ &= f\left[\alpha + \beta\left(E - C_1 - k_1 C_s\right), 1, k_2\right] < f\left[\alpha + \beta\left(E - C_1 - k_1 C_s\right), 1, k_1\right] = U_{\text{base}} . \end{aligned} \tag{D–4}$$

The equality between lines 1 and 2 of equation (D–4) holds because the income terms are identical under a fixed investment budget. The inequality on line 2 holds because $k_2 < k_1$.

Although society is worse off, it may be possible to soften the blow somewhat by increasing the investment budget. This action would reduce the income available for other uses, but yield the benefits of an intermediate number of small projects k_3, where $0 < k_2 < k_3 < k_1$. The result is a utility level:

$$U_{\text{intermediate}} = f\left[\alpha + \beta\left(E - C_2 - k_3 C_s\right), 1, k_3\right].$$

The variable k_3 can generally be chosen so that $U_{\text{intermediate}} > U_{\text{overrun}}$. A small increase in the number of small projects, starting from the level k_2, increases utility as long as:

$$\begin{aligned} 0 \;<\; \frac{\partial}{\partial k} U_{\text{overrun}} \;&=\; \frac{\partial}{\partial k} f\left[\alpha + \beta\left(E - C_2 - k_2 C_s\right), 1, k_2\right] \\ &=\; -\beta C_s f_I + f_k \end{aligned}$$

or:

$$\beta \;<\; f_k / \left(C_s f_I\right).$$

The smaller β, the lower the marginal utility of income, and the more likely this condition is to hold. Thus for smaller β, society is more likely to adjust to the cost overrun by restoring the funding of some small projects, $k_3 > k_2$.

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UNCLASSIFIED

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE
February 1998

3. REPORT TYPE AND DATES COVERED
Final Report, Mar 1997–Feb 1998

4. TITLE AND SUBTITLE
"Probability Distributions of Cost and Sequential Bidding Procedures for Defense Procurement Contracts"

5. FUNDING NUMBERS
DASW01 94 C 0054
T-G7-1353

6. AUTHOR(S)
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Alexandria, VA 22311-1772

8. PERFORMING ORGANIZATION REPORT NUMBER
IDA Document P-3314

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
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OUSD(A&T)
Acquisition Program Integration, Performance Management
The Pentagon, Room 3E1025
Washington, DC 20301

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12A. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12B. DISTRIBUTION CODE

13. ABSTRACT ***(Maximum 200 words)***

This paper explores the use of probability distributions of cost in competitive bidding for government procurement contracts. An attempt is made to identify advantages and disadvantages of using probability distributions, both to the government and to the bidders. The behavioral incentives of all parties are also investigated. A sequential bidding procedure is designed that allows bidders to reveal less information, and the government to process less information, while still inducing honest bidding. The sequential procedure has the same winning bidder, with the same utility to the government and to the bidder, as a one-stage procedure requiring that the bidders furnish and the government evaluate much more information. The paper demonstrates the mathematical equivalence of the one-stage and sequential procedures. It also provides a computer program to execute both procedures and presents a numerical example.

14. SUBJECT TERMS
Government Procurement; Competition; Probability Distribution Functions; Cost Estimates

15. NUMBER OF PAGES
96

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	SAR

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

UNCLASSIFIED